CS 4495 Computer Vision

Camera Model

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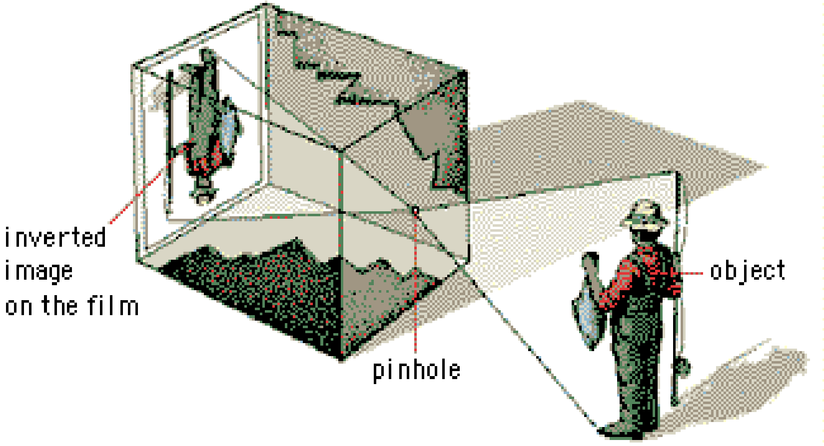


Administrivia

- Problem set 1:
  - How did it go?
  - What would have made it better?
- New problem set will be out by tonight or tomorrow, due Monday, Sept 22^{nd}, 11:55pm
- Today: Camera models and cameras
  - FP Chapter 1 and 2.1-2.2

What is an image?

- Up until now: a function – a 2D pattern of intensity values
- Today: a 2D projection of 3D points

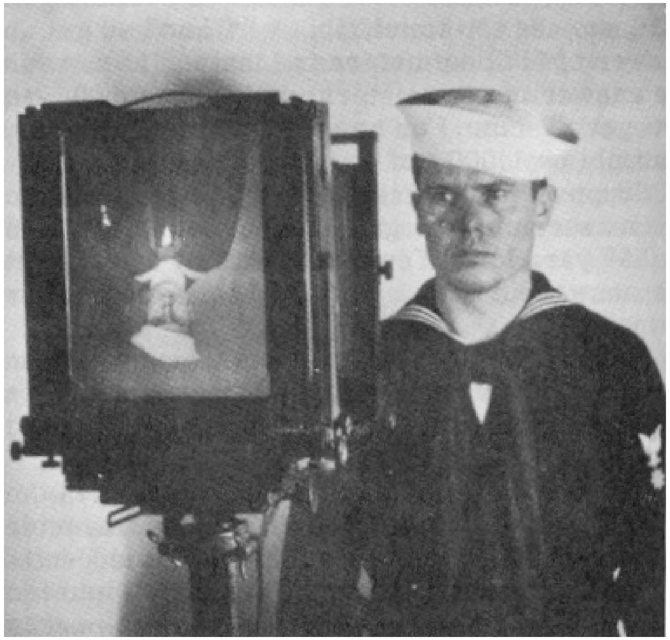
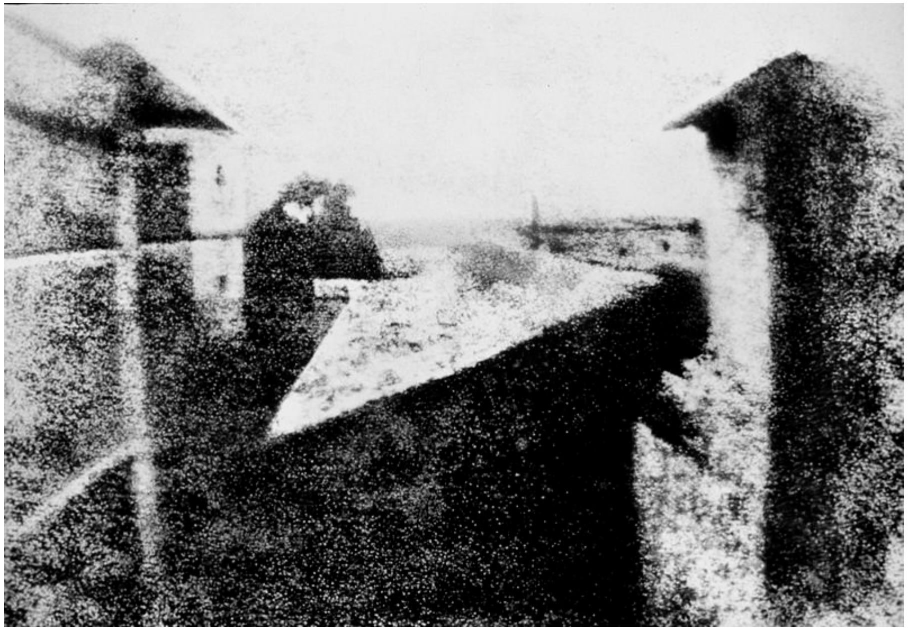


Figure from US Navy Manual of Basic Optics and Optical Instruments, prepared by Bureau of Naval Personnel. Reprinted by Dover Publications, Inc., 1969.

First Known Photograph

View from the Window at le Gras,
Joseph Nicéphore Niépce 1826

Reproduction, 1952



Heliograph- a pewter plate coated with bitumen of Judea (an asphalt derivative of petroleum); after at least a day-long exposure of eight hours, the plate was removed and the latent image of the view from the window was rendered visible by washing it with a mixture of oil of lavender and white petroleum which dissolved away the parts of the bitumen which had not been hardened by light. – Harry Ransom Center UT Austin

What is a camera/imaging system?

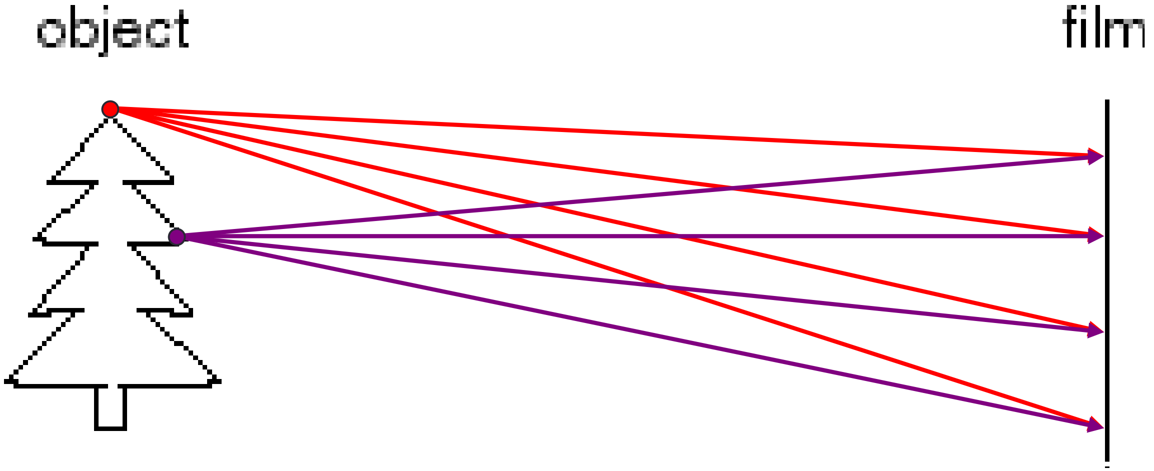
- Some device that allows the projection of light from 3D points to some “medium” that will record the light pattern.
- A key to this is “projection”…

Projection


MAKE POVERTY HISTORY
JULIAN BEEVER
CoolOpticalIllusions.com

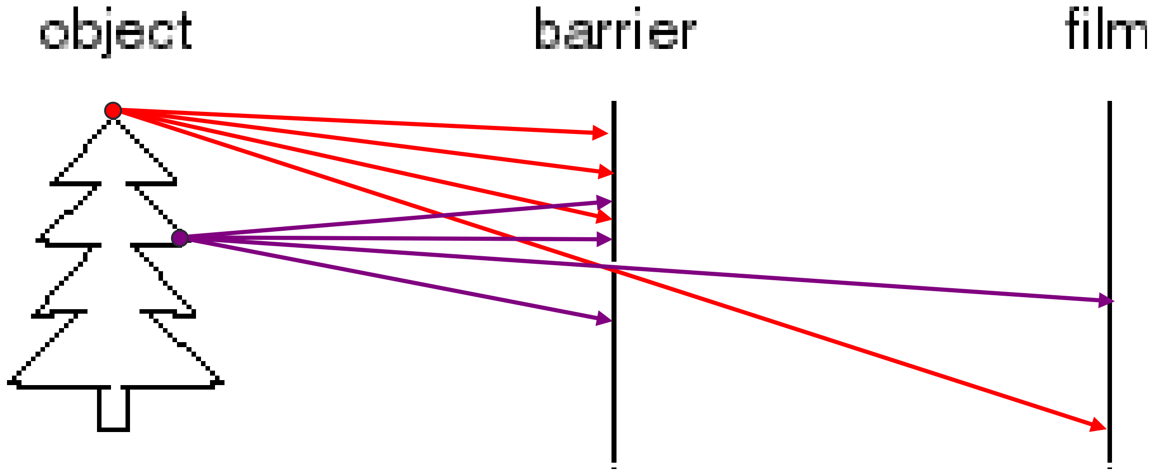
Projection

Image formation



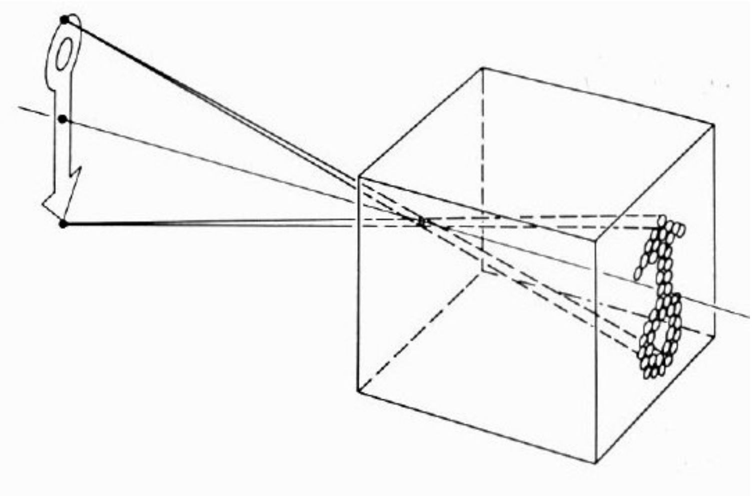
- Let's design a camera
  - Idea 1: put a piece of film in front of an object
  - Do we get a reasonable image?

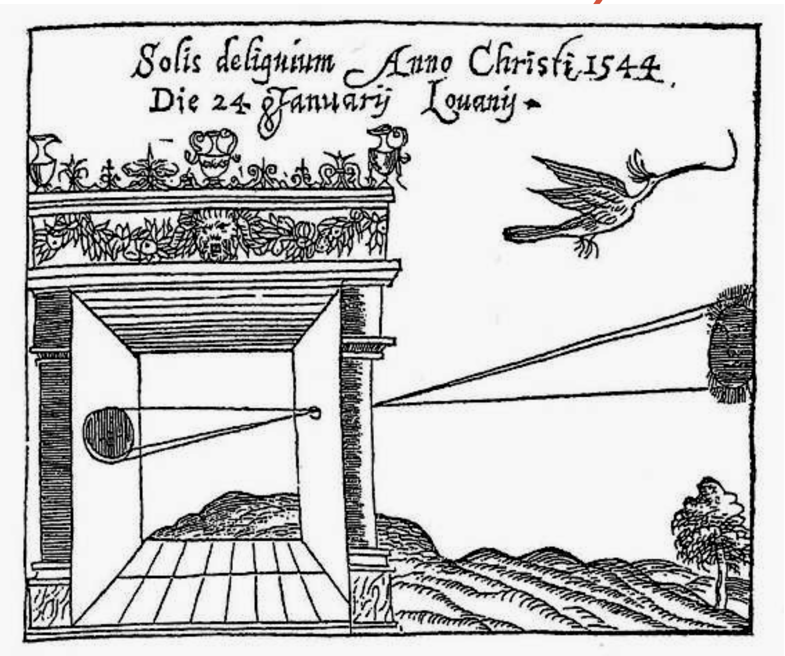
Pinhole camera



- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening known as the **aperture**
  - How does this transform the image?

Camera Obscura (Latin: Darkened Room)





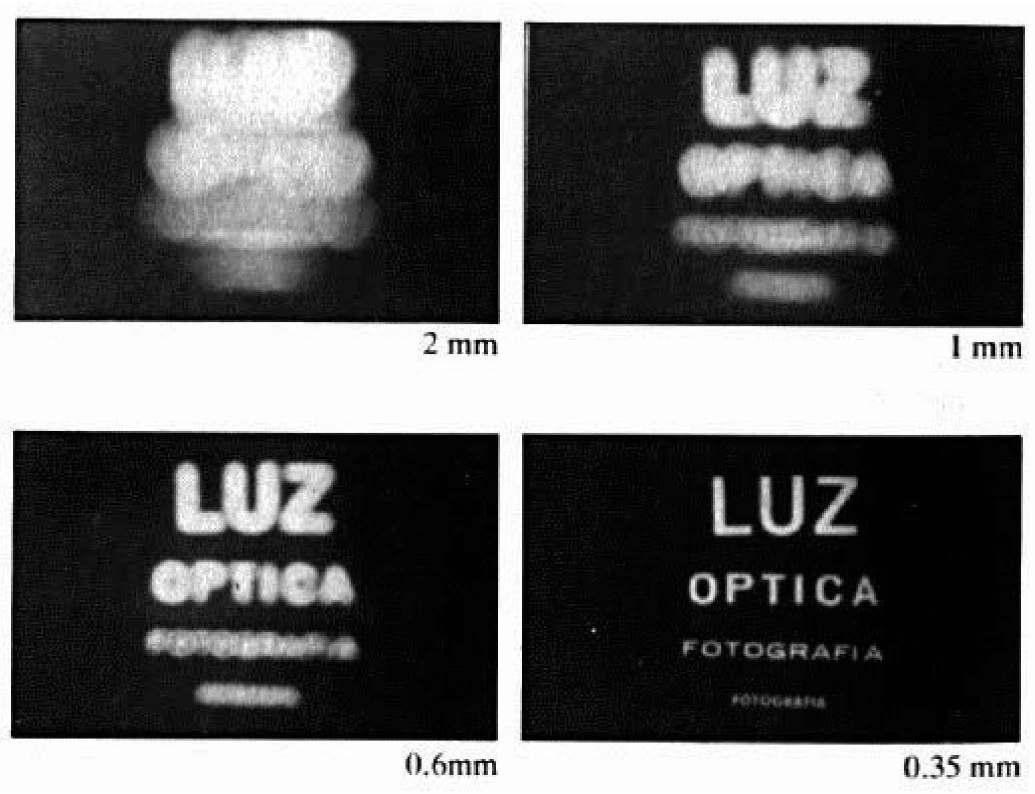
- The first camera
  - Known to Aristotle (384-322 BCE)
  - According to DaVinci “When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size, in a reversed position, owing to the intersection of the rays".
  - Depth of the room is the “focal length”
  - How does the aperture size affect the image?

Home-made pinhole camera

Why so blurry?

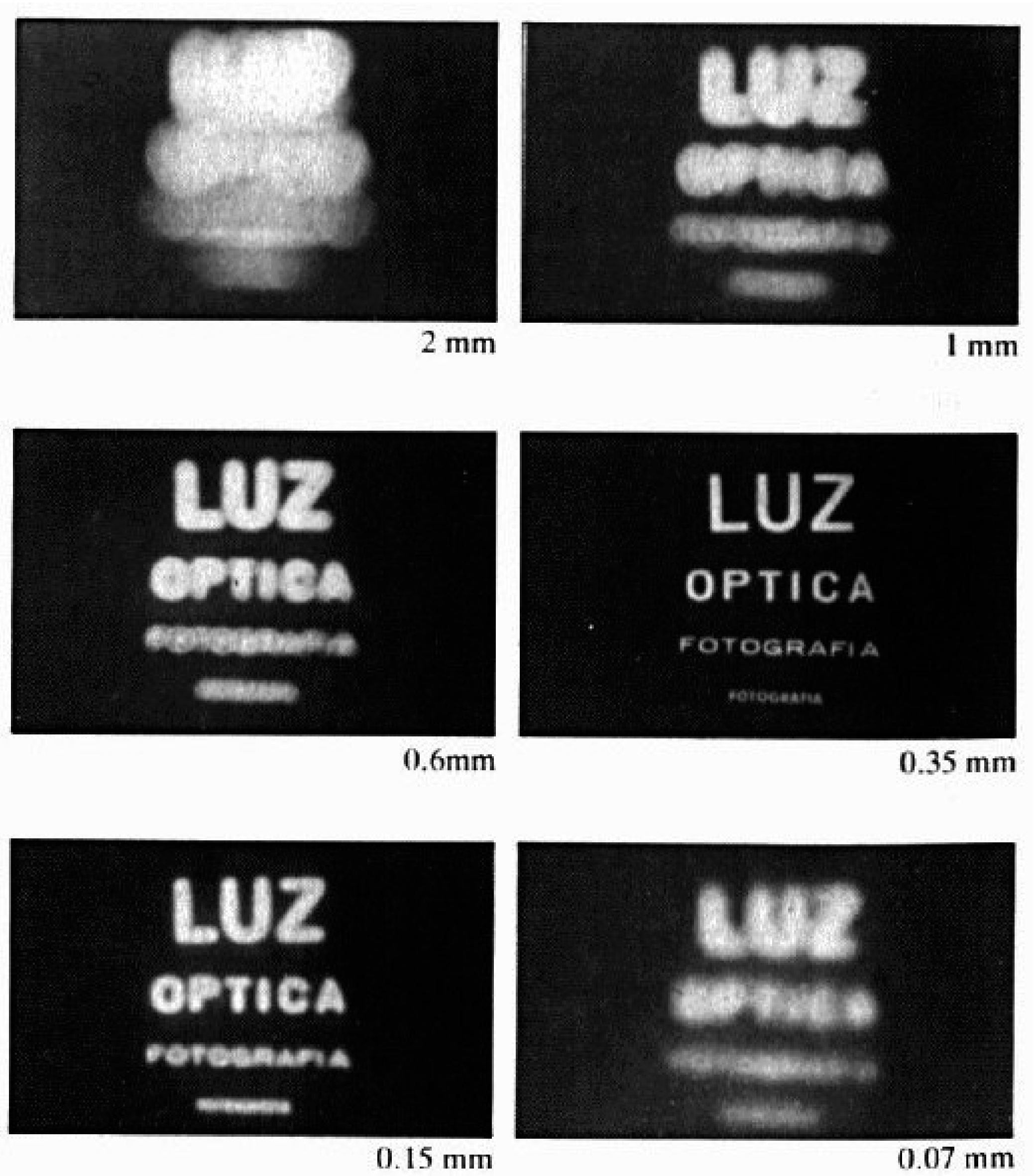
http://www.debevec.org/Pinhole/

Shrinking the aperture

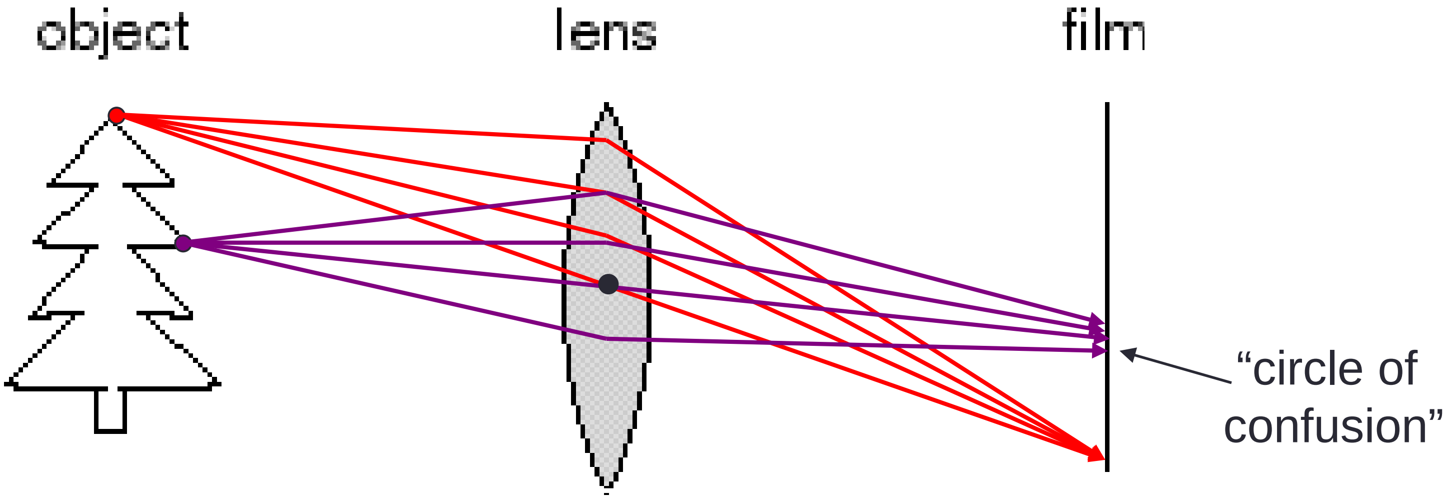


- Why not make the aperture as small as possible?
  - Less light gets through
  - *Diffraction* effects…

Shrinking the aperture

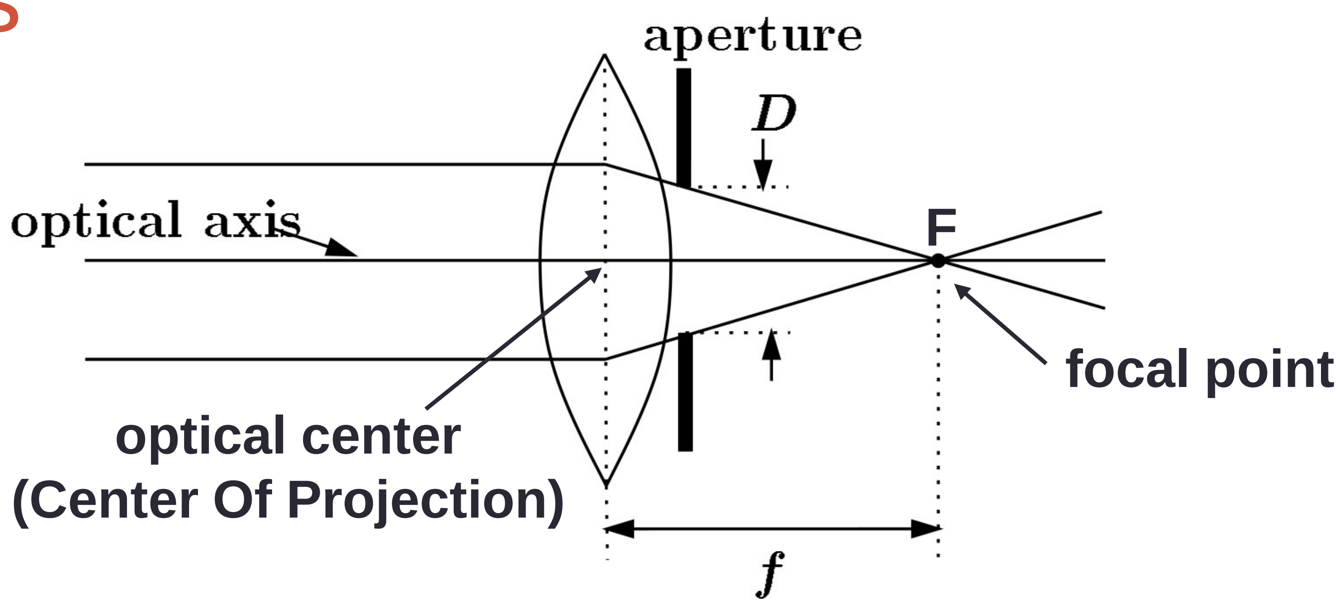


Adding a lens – and concept of focus



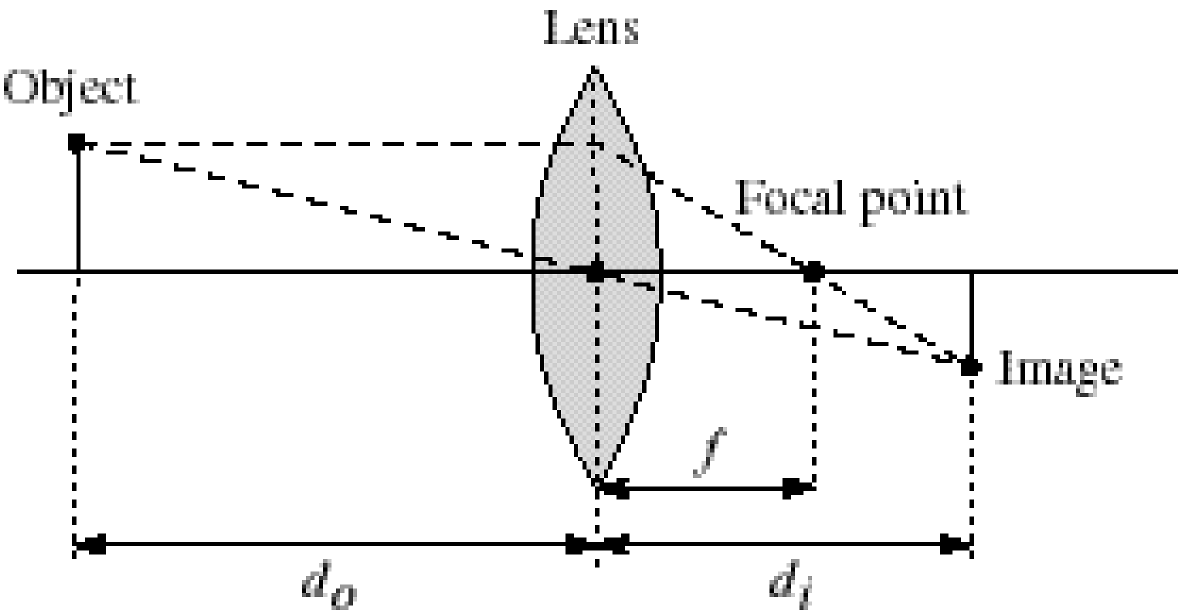
- A lens focuses light onto the film
  - There is a specific distance at which objects are “in focus”
    - other points project to a “circle of confusion” in the image
  - Changing the shape of the lens changes this distance

Lenses



- A lens focuses parallel rays onto a single focal point
  - focal point at a distance *f* beyond the plane of the lens
    - *f* is a function of the shape and index of refraction of the lens
  - Aperture of diameter D restricts the range of rays
    - aperture may be on either side of the lens
  - Lenses used to be typically spherical (easier to produce) but now many “aspherical” elements – designed to improve variety of “aberrations”.

Thin lenses

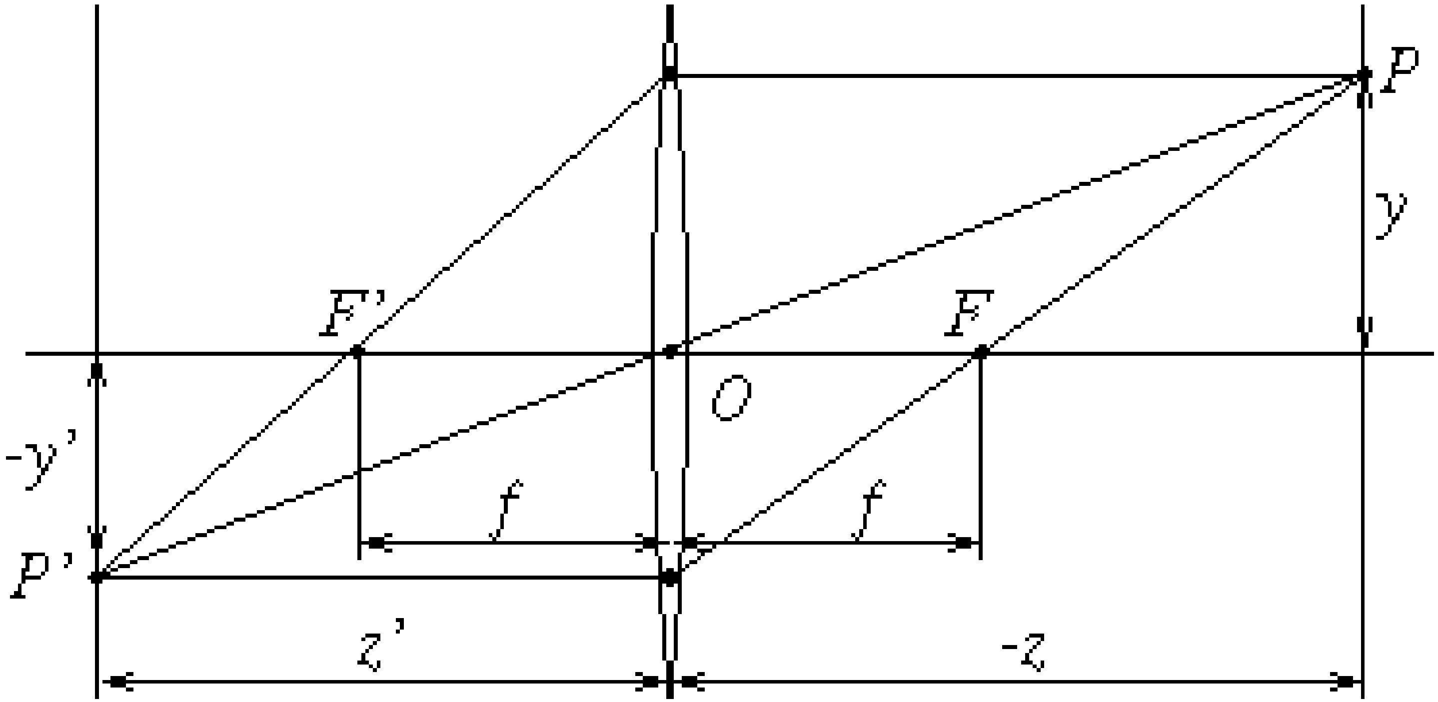


Thin lens model assumes thickness is small compared to curvature:

1. Any ray parallel to the axis on one side of the lens passes through the focal point on the other side.
2. Any ray that passes through the center of the lens will not change its direction.

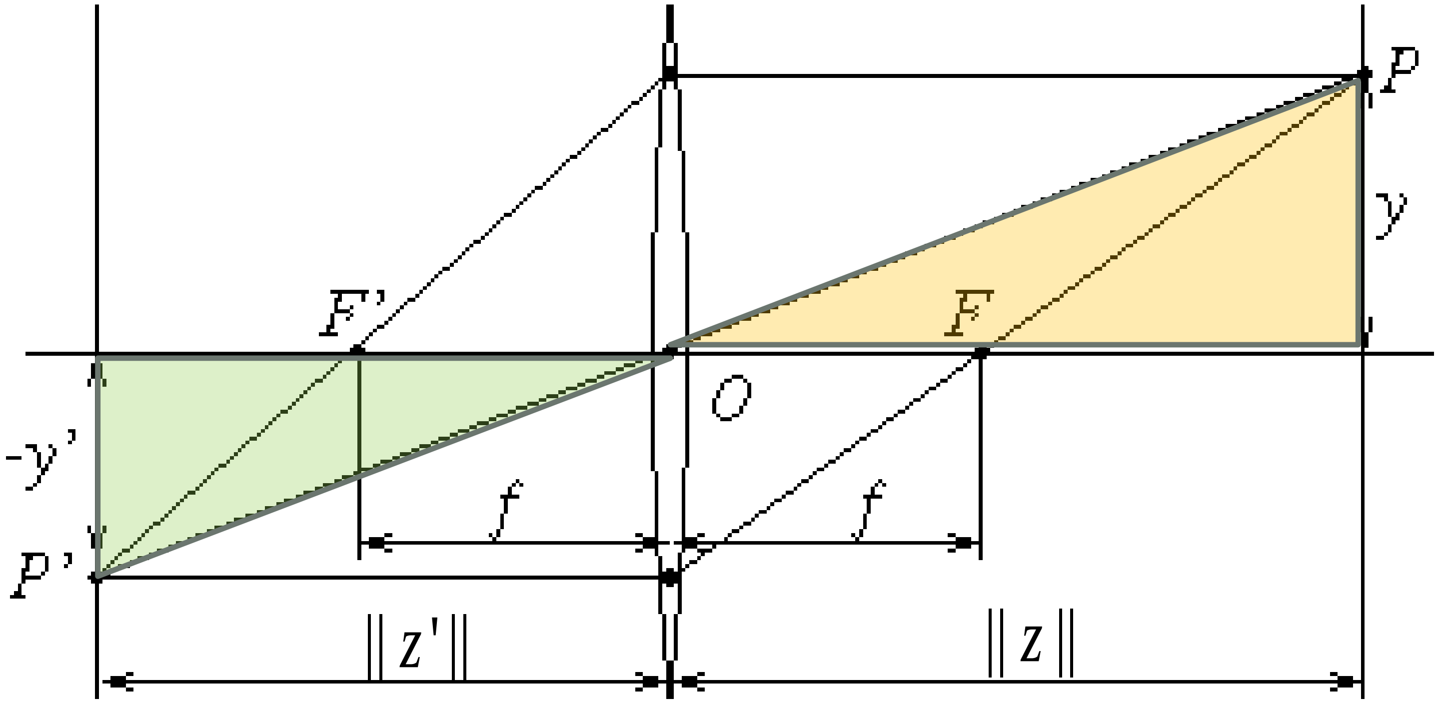
This gives rise to the “thin lens equation”…

The thin lens equation



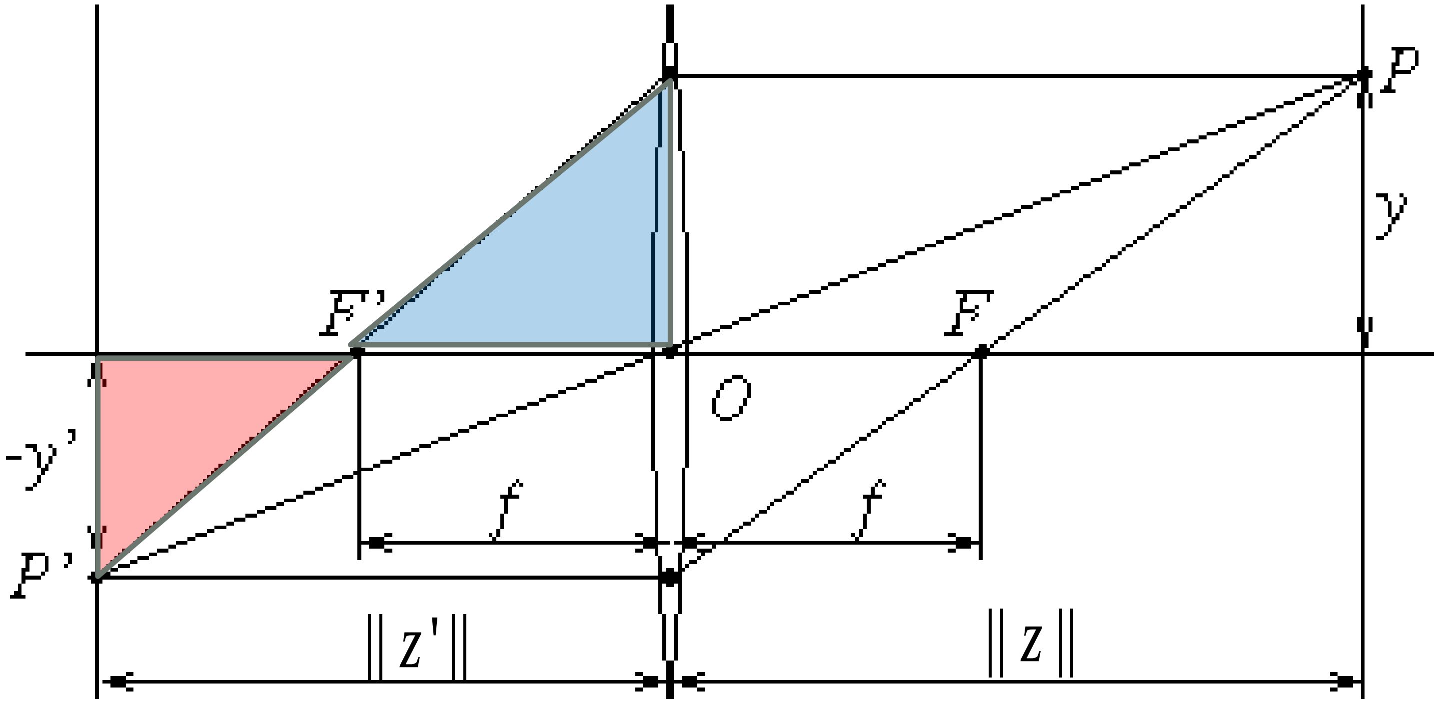
Computer Vision - A Modern Approach
Set: Cameras
Slides by D.A. Forsyth

The thin lens equation



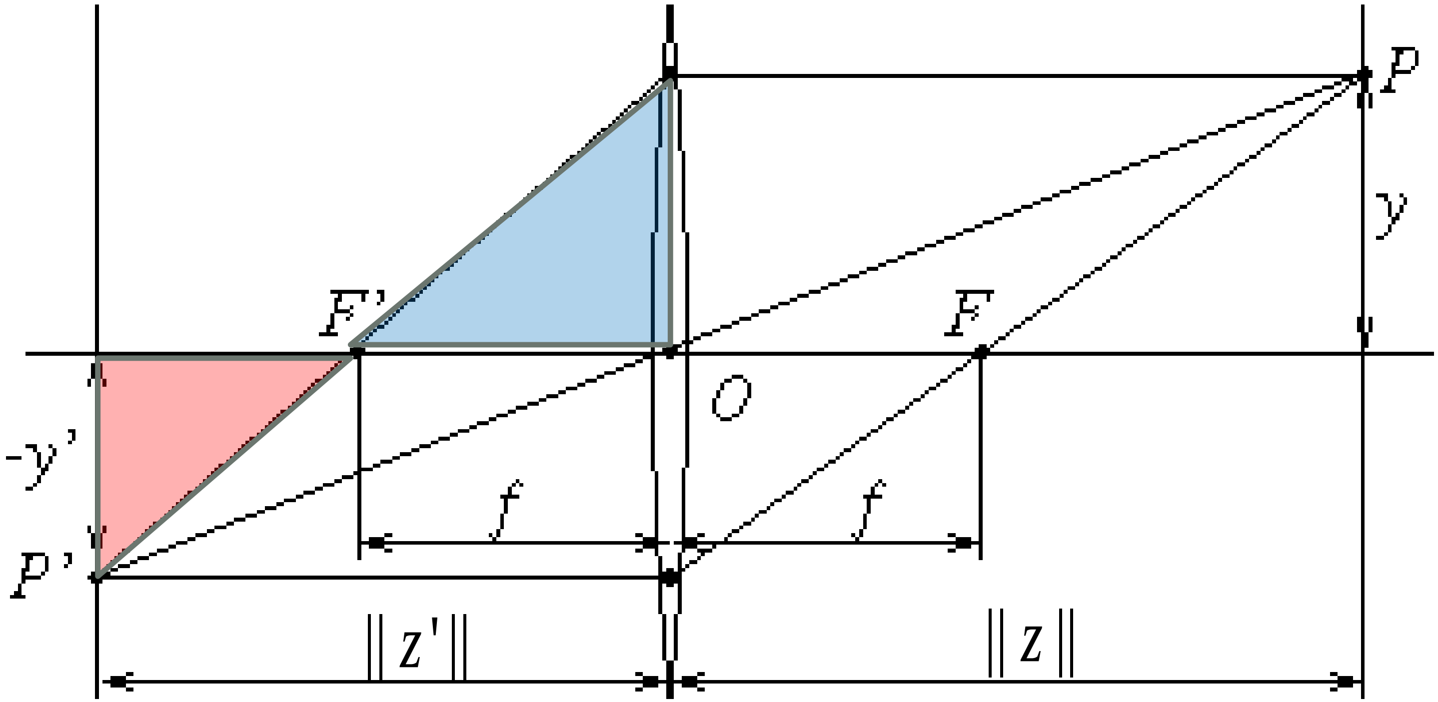
$$\frac{-y'}{y} = \frac{\| z' \|}{\| z \|}$$

The thin lens equation



$$\frac{-y'}{y} = \frac{\|z'\|}{\|z\|} \qquad \frac{-y'}{y} = \frac{\|z'\|-f}{f} \qquad \boxed{\rightarrow \frac{\|z'\|}{\|z\|} = \frac{\|z'\|-f}{f}}$$

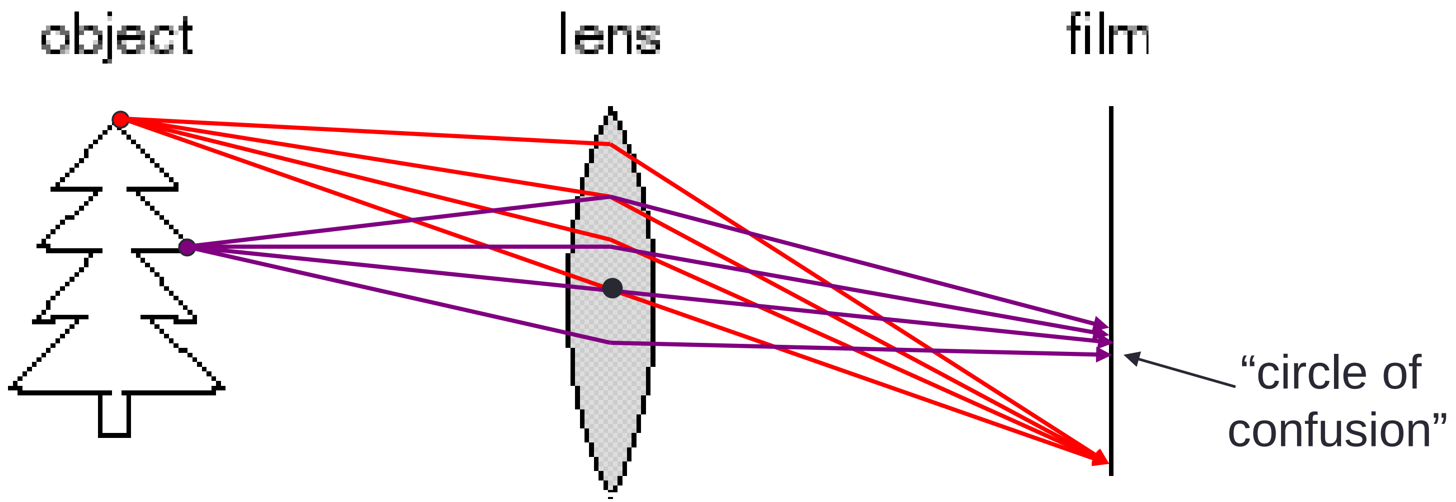
The thin lens equation



$$\frac{\|z'\|}{\|z\|} = \frac{\|z'\| - f}{f} \rightarrow \frac{1}{\|z\|} = \frac{1}{f} - \frac{1}{\|z'\|} \rightarrow \boxed{\frac{1}{\|z'\|} + \frac{1}{\|z\|} = \frac{1}{f}}$$

Thin lens equation

What's in focus and what's not?

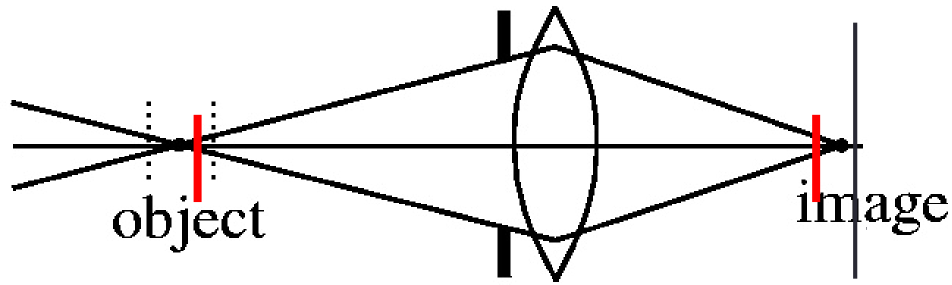


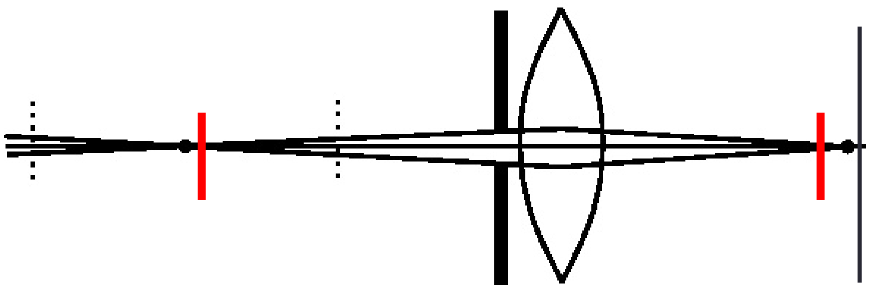
- A lens focuses light onto the film
  - There is a **specific distance** at which objects are "in focus"
    - other points project to a "circle of confusion" in the image
  - Changing the shape or relative locations of the lens elements changes this distance

Varying Focus

Ren Ng

Depth of field





f / 5.6

f / 32

- Changing the aperture size affects depth of field
  - A smaller aperture increases the range in which the object is approximately in focus
  - Aside: could actually compute distance from defocus
  - But small aperture reduces amount of light – need to increase exposure

Flower images from Wikipedia http://en.wikipedia.org/wiki/Depth_of_field

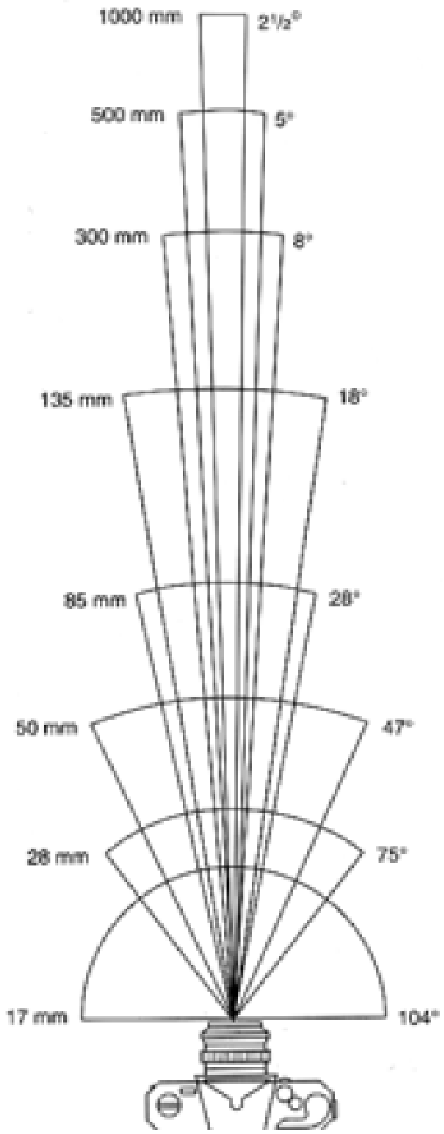
Varying the aperture

Large apeture = small DOF

Small apeture = large DOF

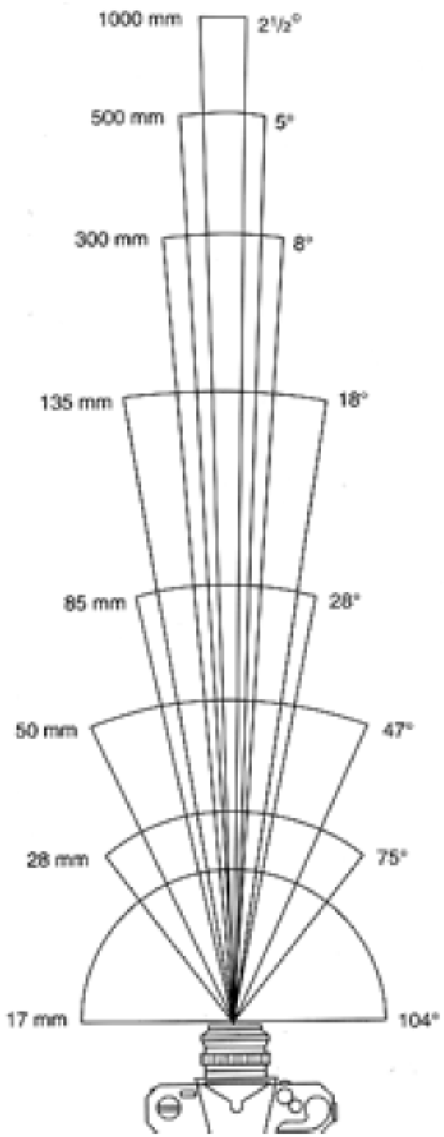
Nice Depth of Field effect

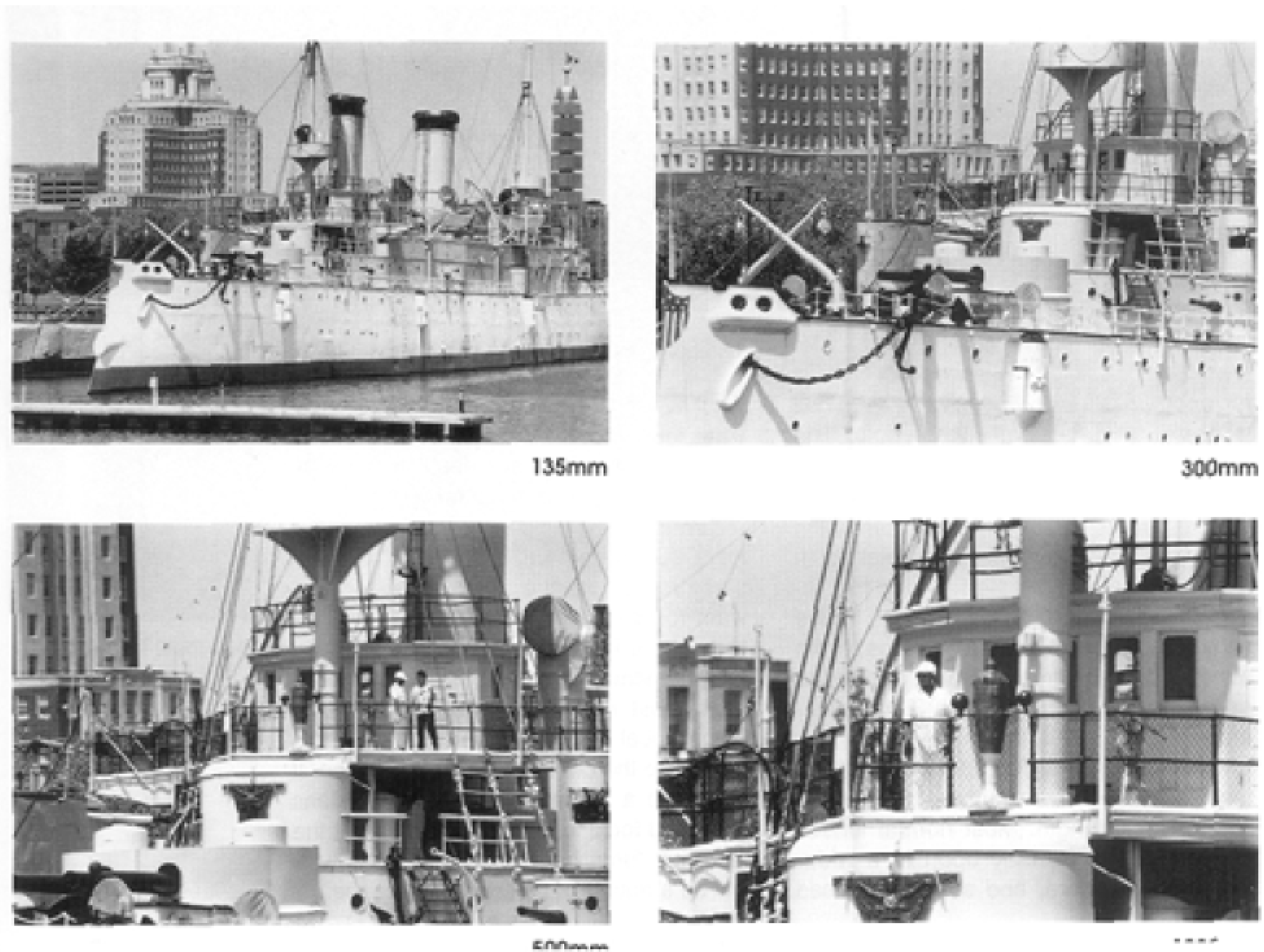
Field of View (Zoom)



From London and Upton

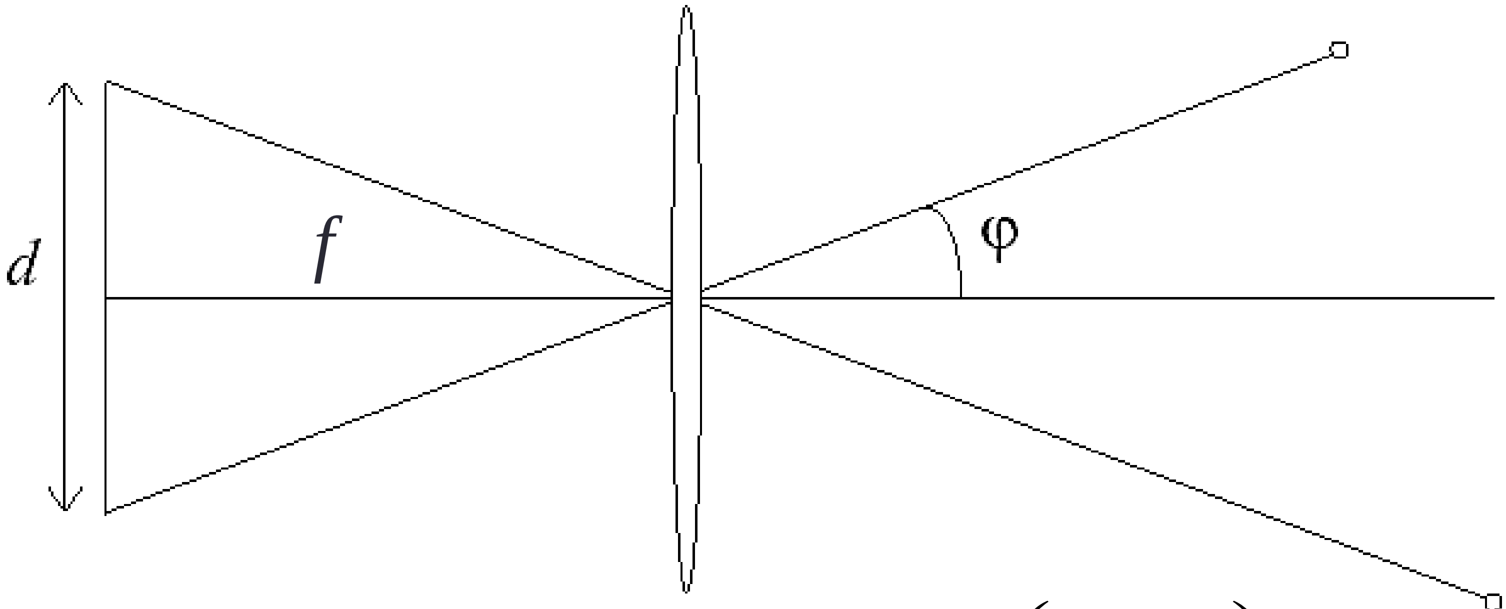
Field of View (Zoom)





From London and Upton

FOV depends on Focal Length



d is the "retina" or sensor size

$$\phi = 2\tan^{-1}\left(\frac{d/2}{f}\right)$$

Larger Focal Length => Smaller FOV

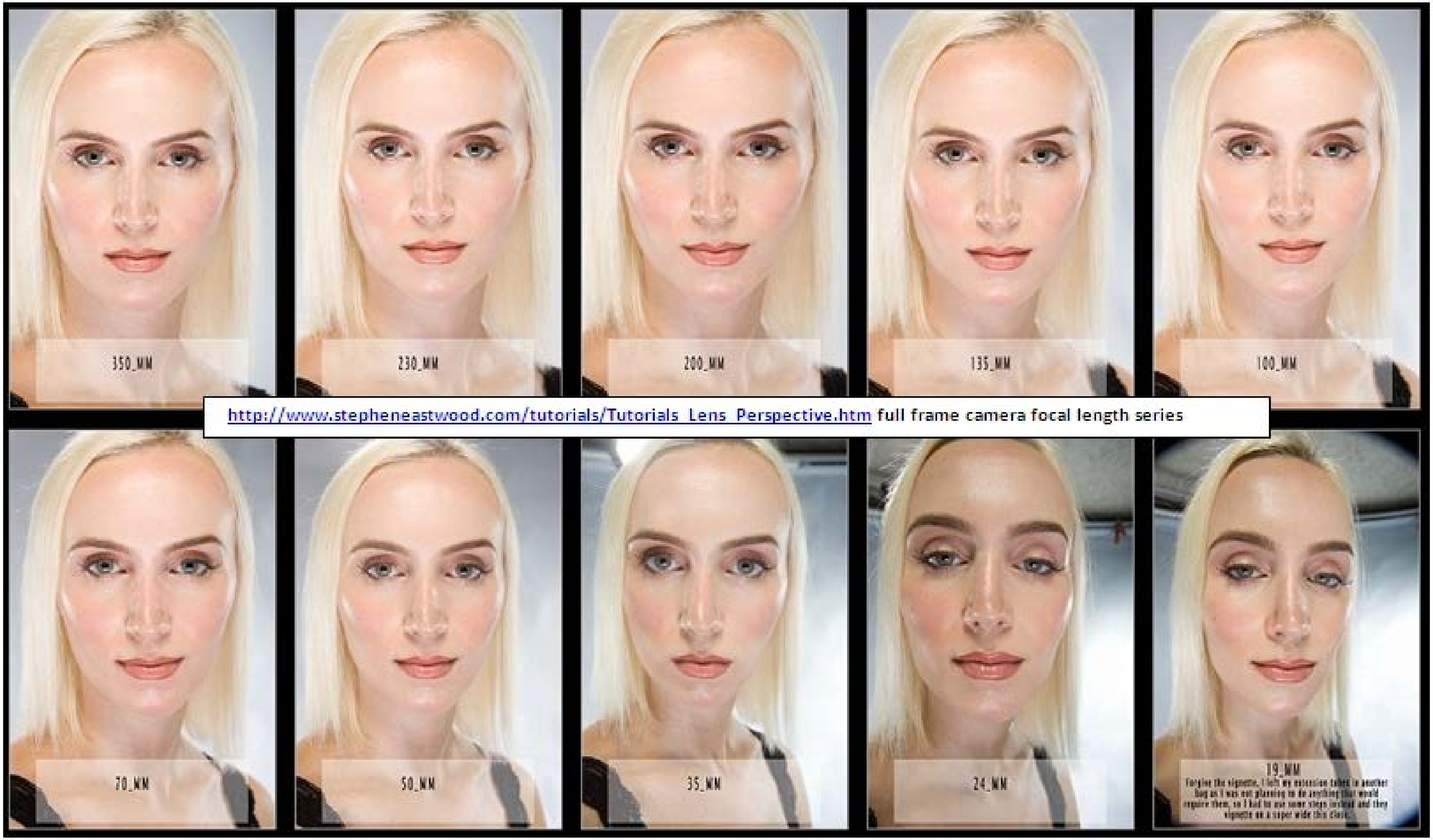
Zooming and Moving are not the same…

Field of View / Focal Length

Large FOV, small f
Camera close to car

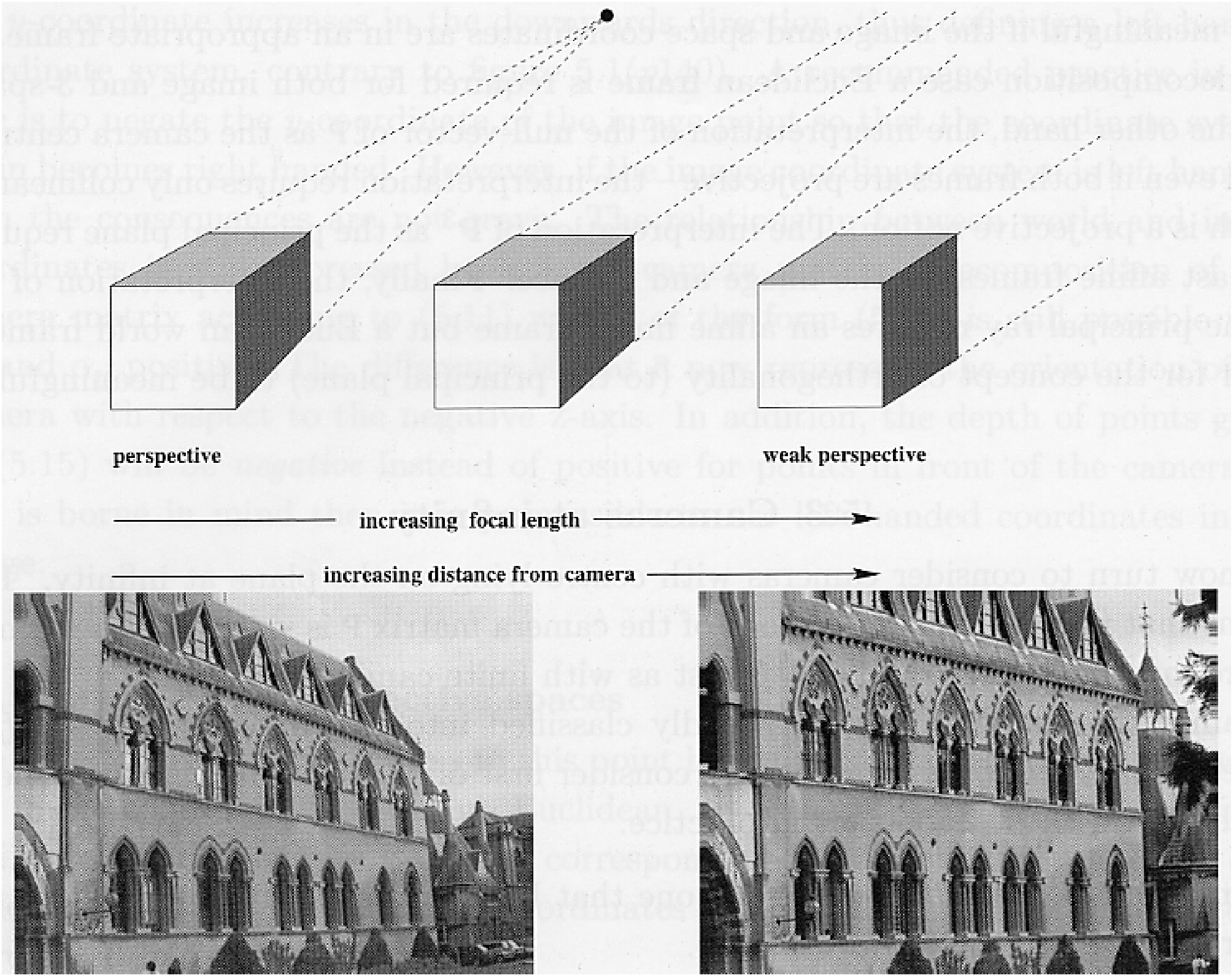
Small FOV, large f
Camera far from the car

Perspective and Portraits



Perspective and Portraits

Effect of focal length on perspective effect



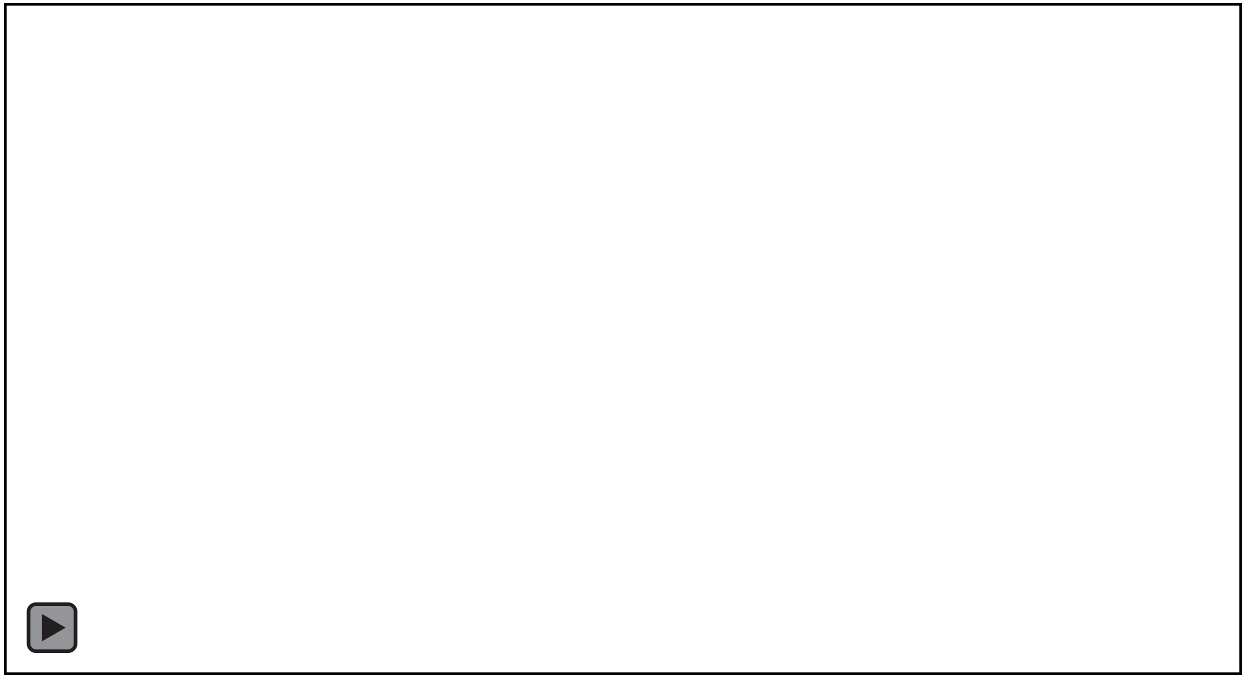
From Zisserman & Hartley

Dolly Zoom

- Move camera while zooming, keeping foreground stationary
- Pioneered by Hitchcock in *Vertigo* (1958)
- Original(YouTube link) (2:07)

Dolly Zoom – Hitchcock's *Vertigo*

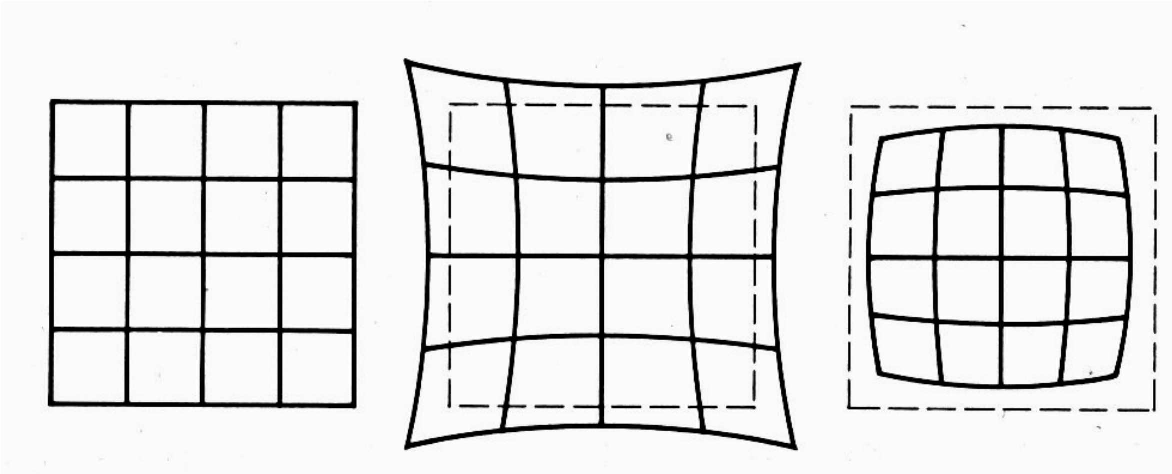
Dolly Zoom – “anyone can do it”



But reality can be a problem…

- Lenses are not thin
- Lenses are not perfect
- Sensing arrays are almost perfect
- Photographers are not perfect – except some of us…

Geometric Distortion



No distortion Pin cushion Barrel

- Radial distortion of the image
  - Caused by imperfect lenses
  - Deviations are most noticeable for rays that pass through the edge of the lens

Modeling geometric distortion

Assume project $(\widehat{x}, \widehat{y}, \widehat{z})$ to "normalized" image coordinates

$$x'_n = \widehat{x}/\widehat{z}$$
$$y'_n = \widehat{y}/\widehat{z}$$

Apply radial distortion

$$r^2 = {x'_n}^2 + {y'_n}^2$$
$$x'_d = x'_n(1 + \kappa_1 r^2 + \kappa_2 r^4)$$
$$y'_d = y'_n(1 + \kappa_1 r^2 + \kappa_2 r^4)$$

Apply focal length translate image center

$$x' = f x'_d + x_c$$
$$y' = f y'_d + y_c$$

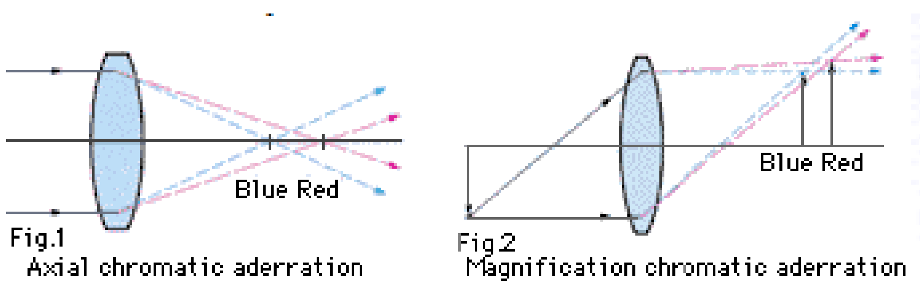
- To model lens distortion
  - Use above projection operation instead of standard projection matrix multiplication *(which you haven't seen yet!)*

Correcting radial distortion

from [Helmut Dersch]

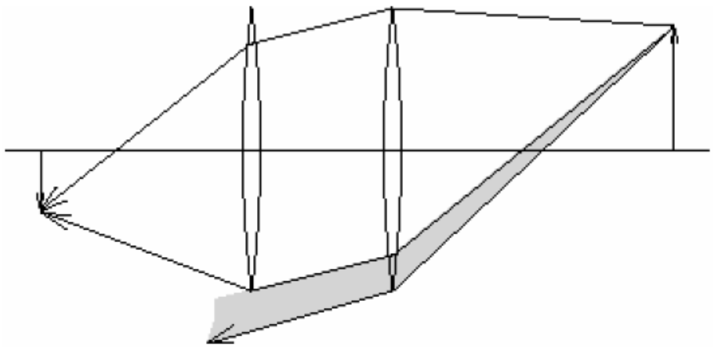
Chromatic Aberration

Rays of different wavelength focus in different planes



Can be significantly improved by "undistorting" each channel separately

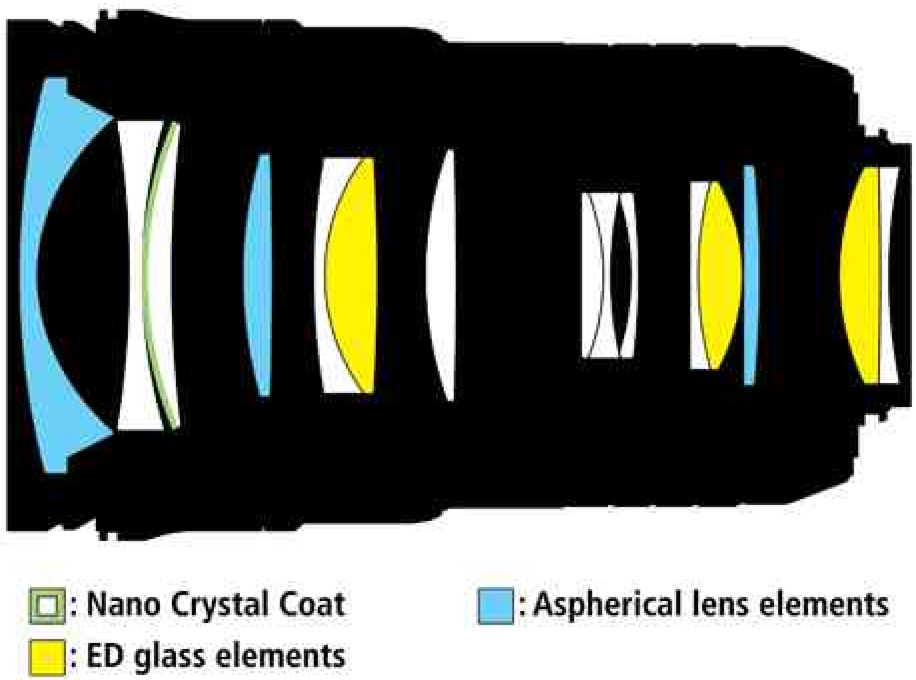
Vignetting



- Some light misses the lens or is otherwise blocked by parts of the lens

Lens systems

Nikon 24-70mm zoom



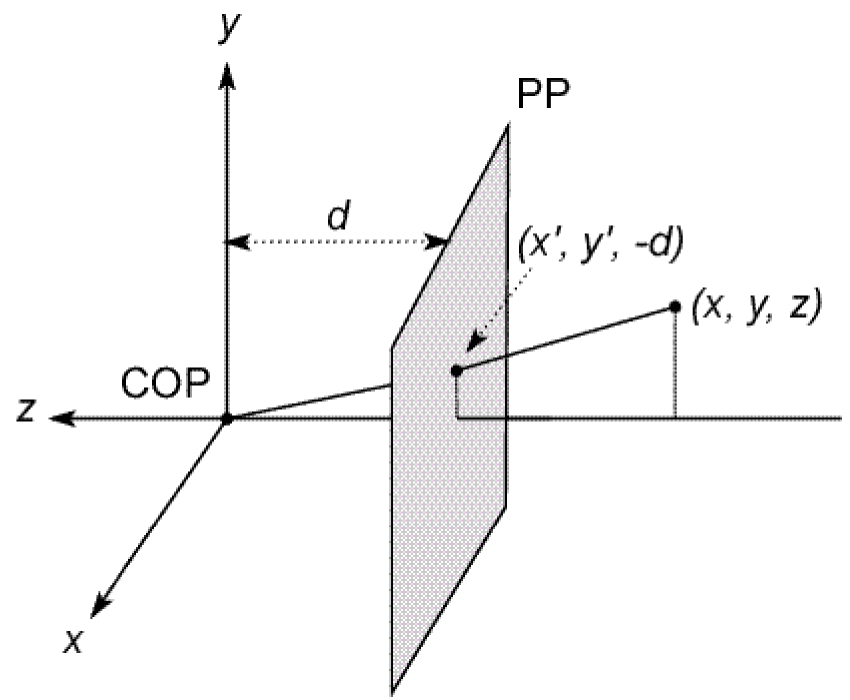
- Real lenses combat these effects with multiple elements.
- Computer modeling has made lenses lighter and better.
- Special glass, aspherical elements, etc.

Retreat to academia!!!

- We will assume a pinhole model
- No distortion (yet)
- No aberrations

Modeling projection – coordinate syste

- We will use the pin-hole model as an approximation
- Put the optical center (**C**enter **O**f **P**rojection) at the origin
- STANDARD (x,y) COORDINATE SYSTEM
- Put the image plane (**P**rojection **P**lane) *in front* of the COP
  - Why?
- The camera looks down the *negative* z axis
  - we need this if we want right-handed-coordinates

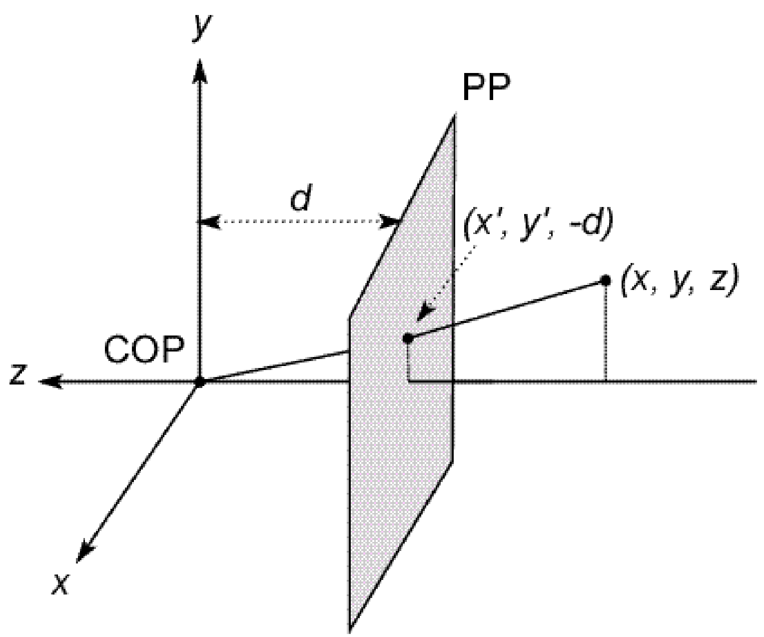


Modeling projection

- Projection equations
  - Compute intersection with *Perspective Projection* of ray from (x,y,z) to COP
  - Derived using similar triangles

Distant objects are smaller

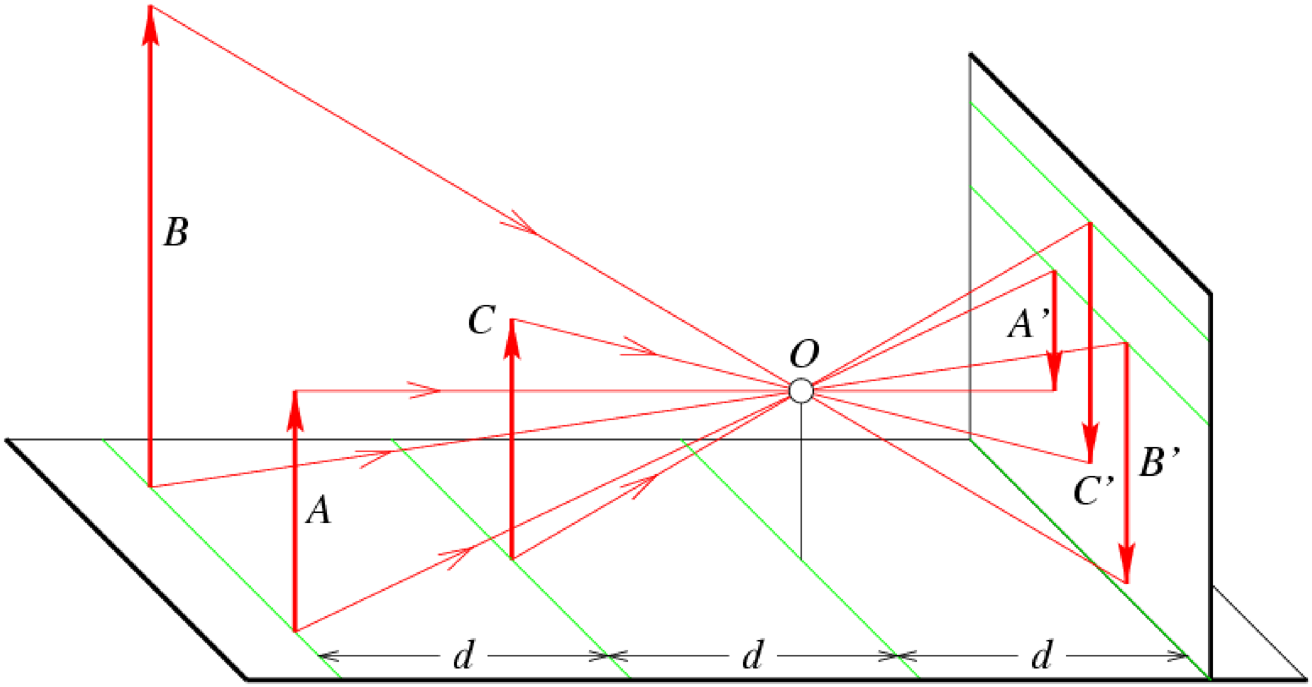
$$(x, y, z) \rightarrow (-d\frac{x}{z},\ -d\frac{y}{z},\ -d)$$



- We get the projection by throwing out the last coordinate:

$$(x, y, z) \rightarrow (-d\frac{x}{z},\ -d\frac{y}{z})$$

Distant objects appear smaller



Homogeneous coordinates

- Is this a linear transformation?
  - No – division by Z is non-linear

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad (x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous image (2D) coordinates

homogeneous scene (3D) coordinates

Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w) \qquad \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

Homogenous coordinates invariant under scale

Perspective Projection

- Projection is a matrix multiply using homogeneous coordinates:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f \end{bmatrix} \Rightarrow \left(f\frac{x}{z}, f\frac{y}{z} \right)$$

$$\Rightarrow (u, v)$$

This is known as perspective projection

- The matrix is the projection matrix
- The matrix is only defined up to a scale
- *f* is for “focal length – used to be *d*

Perspective Projection

- How does scaling the projection matrix change the transformation?

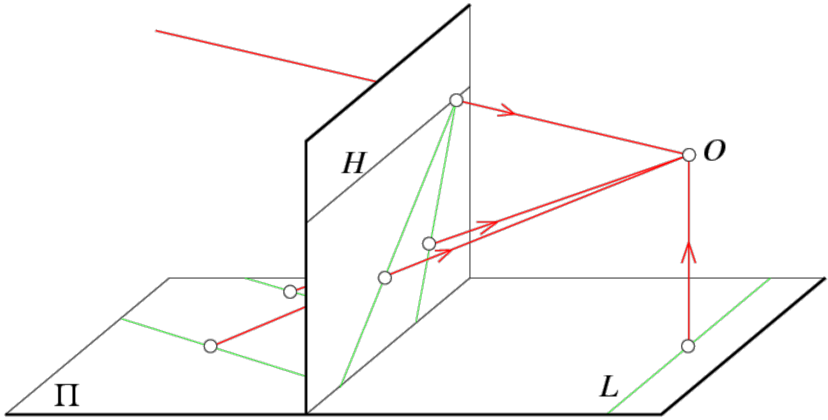
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z/f \end{bmatrix} \Rightarrow \left(f\frac{x}{z}, f\frac{y}{z} \right)$$

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} fx \\ fy \\ z \end{bmatrix} \Rightarrow \left(f\frac{x}{z}, f\frac{y}{z} \right)$$

S. Seitz

Geometric properties of projection

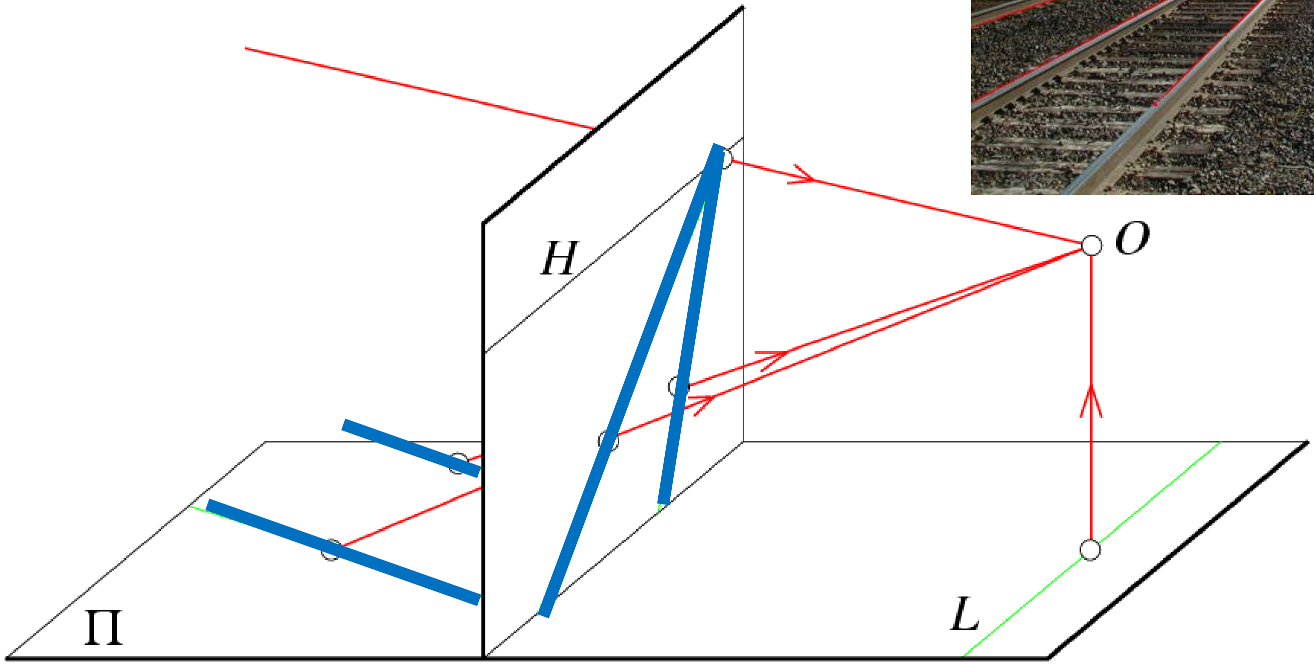
- Points go to **points**
- Lines go to **lines**
- Polygons go to **polygons**
- Planes go to **planes** (or half planes)



- Degenerate cases:
  - line in the world through focal point yields **point**
  - plane through focal point yields **line**

Parallel lines in the world meet in the image

- “**Vanishing**” point



Parallel lines converge in math too…

Line in 3-space

$$x(t) = x_0 + at$$

$$y(t) = y_0 + bt$$

$$z(t) = z_0 + ct$$

Perspective projection of the line

$$x'(t) = \frac{fx}{z} = \frac{f(x_0 + at)}{z_0 + ct}$$

$$y'(t) = \frac{fy}{z} = \frac{f(y_0 + bt)}{z_0 + ct}$$

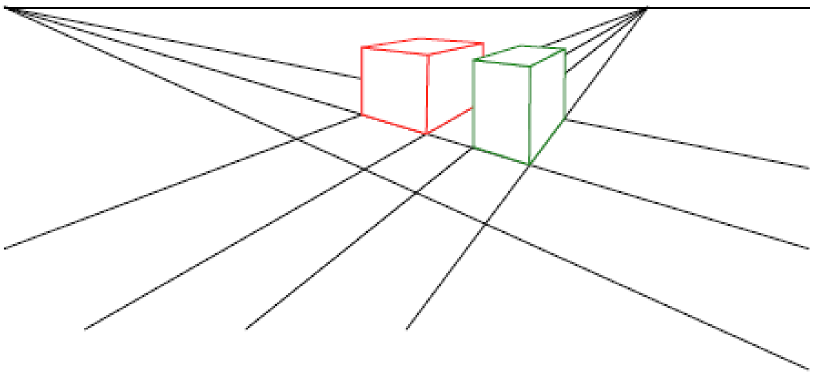
In the limit as $t \to \pm\infty$ we have (for $c \neq 0$):

$$x'(t) \to \frac{fa}{c}, \quad y'(t) \to \frac{fb}{c}$$

This tells us that any set of parallel lines (same a, b, c parameters) project to the same point (called the vanishing point). What does it mean if c=0?

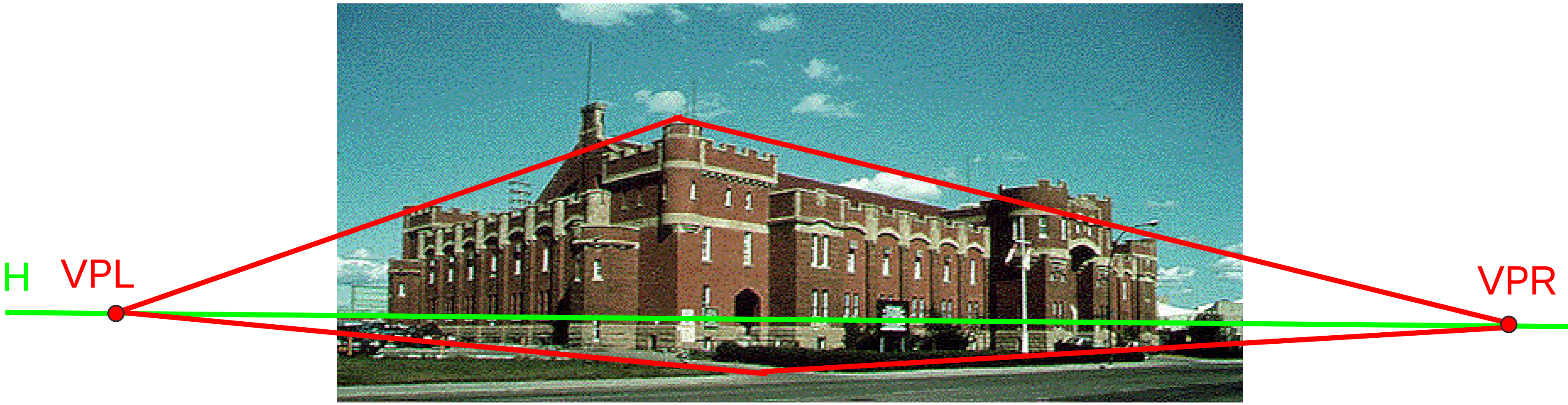
Vanishing points

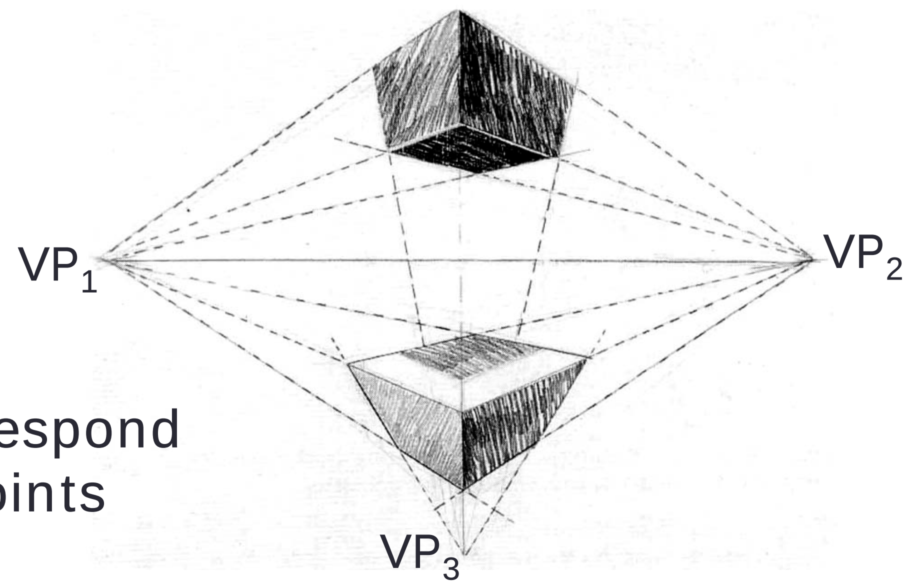
- Each set of parallel lines (=direction) meets at a different point
  - The *vanishing point* for this direction
- Sets of parallel lines on the same plane lead to *collinear* vanishing points.
  - The line is called the *horizon* for that plane



- Good ways to spot faked images
  - scale and perspective don't work
  - vanishing points behave badly
  - supermarket tabloids are a great source.

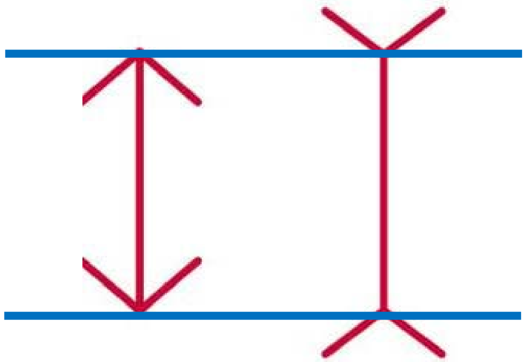
Vanishing points



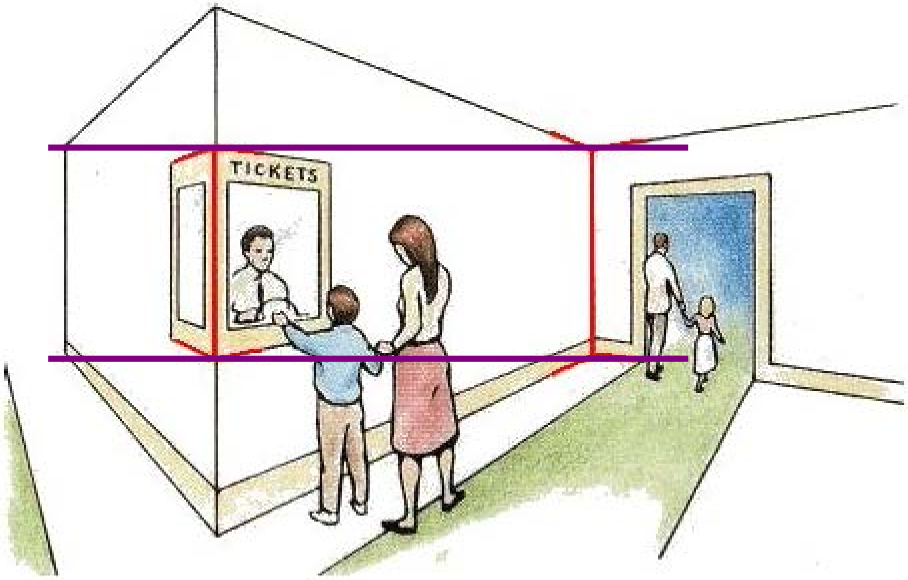


Different directions correspond to different vanishing points

Human vision: Müller-Lyer Illusion

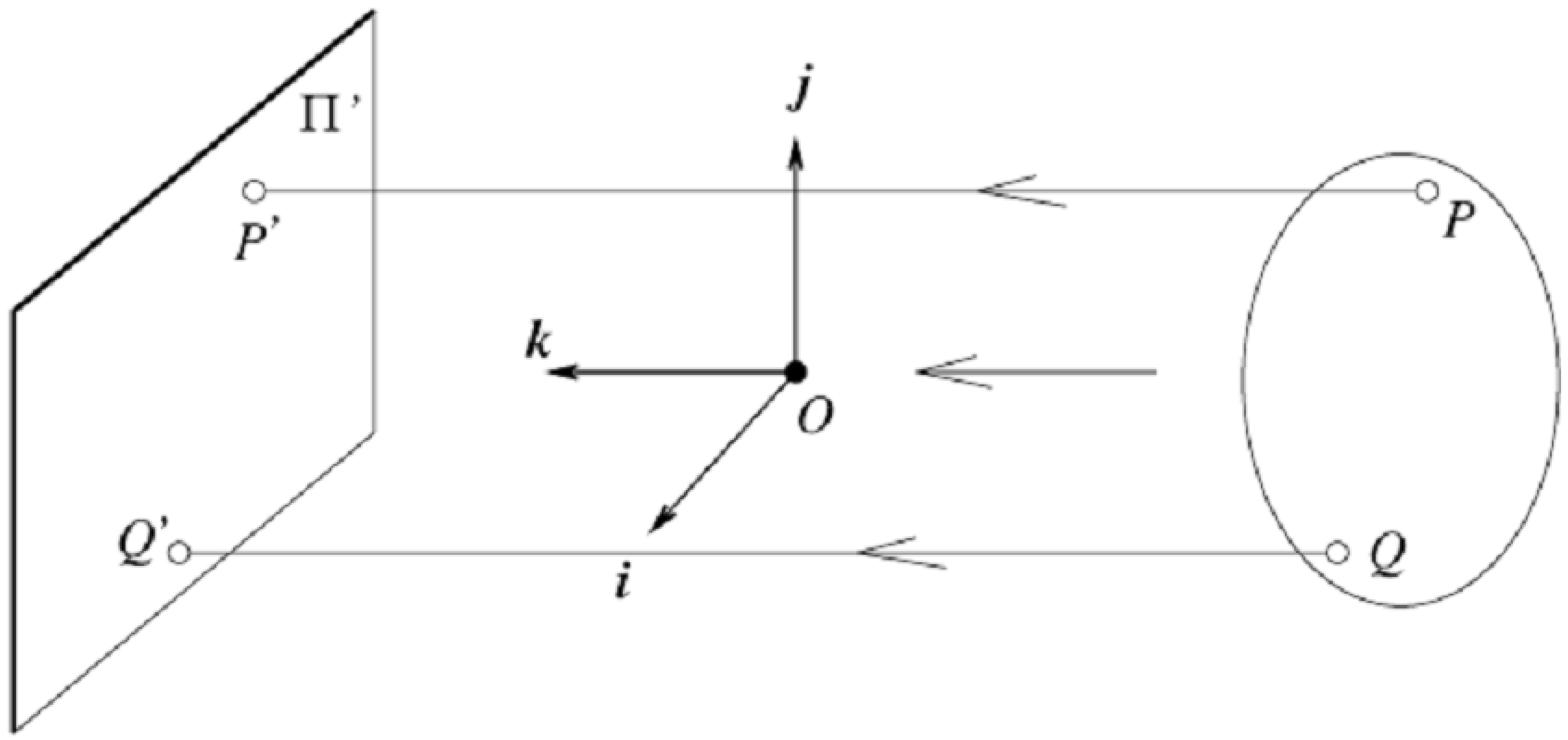


Which line is longer?



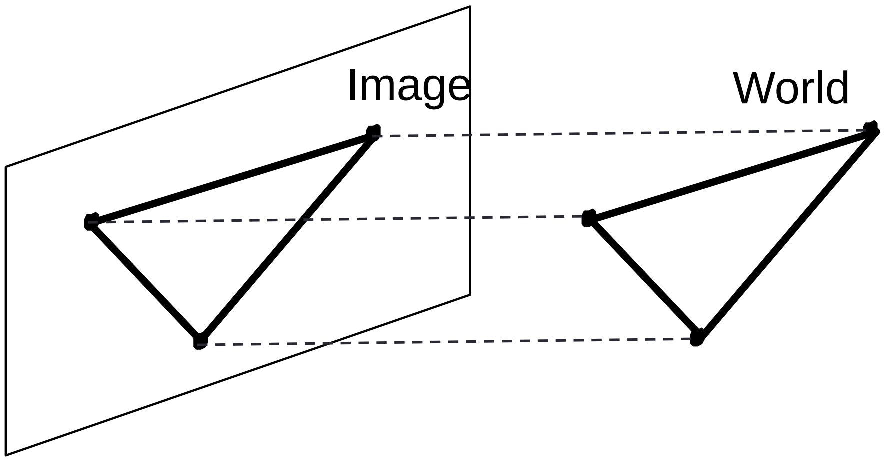
http://www.michaelbach.de/ot/sze_muelue/index.html

Other projection models: Orthographic projection



Orthographic projection

- Special case of perspective projection
  - Distance from the COP to the PP is infinite

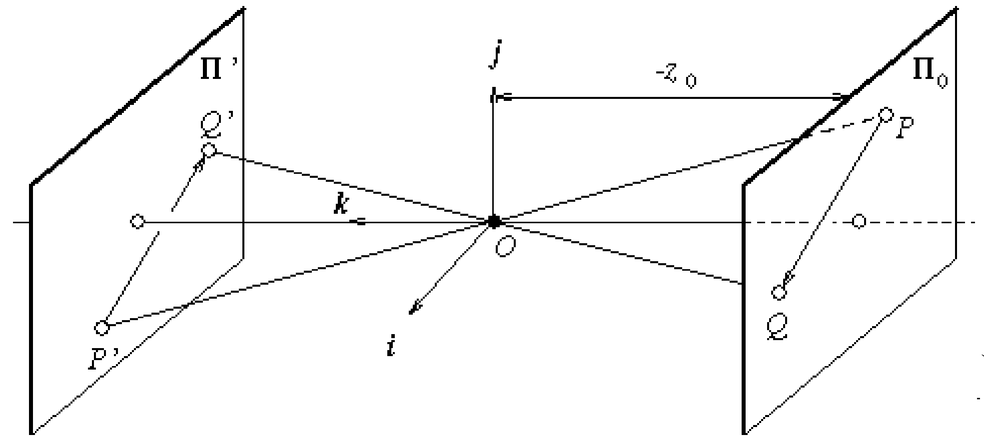


  - Good approximation for telephoto optics
  - Also called "parallel projection": (x, y, z) → (x, y)
  - What's the projection matrix?

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow (x, y)$$

Other projection models: Weak perspective

- Issue
  - Perspective effects, but not over the scale of individual objects
  - Collect points into a group at about the same depth, then divide each point by the depth of its group
  - Adv: easy
  - Disadv: only approximate



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1/s \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1/s \end{bmatrix} \Rightarrow (sx, sy)$$

Three camera projections

3-d point **2-d image position**

(1) Perspective: $$(x, y, z) \rightarrow \left(\frac{fx}{z}, \frac{fy}{z} \right)$$

(2) Weak perspective: $$(x, y, z) \rightarrow \left(\frac{fx}{z_0}, \frac{fy}{z_0} \right)$$

(3) Orthographic: $$(x, y, z) \rightarrow (x, y)$$